

Current reversals in an inhomogeneous system with asymmetric unbiased fluctuations

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Abstract. We present a study of the transport of a Brownian particle moving in a periodic symmetric potential in the presence of asymmetric unbiased fluctuations. The particle is considered to move in a medium with periodic space dependent friction. By tuning the parameters of the system, the direction of the current exhibits reversals, both as a function of temperature as well as the amplitude of rocking force. We found that the mutual interplay between the opposite driving factors is the necessary term for current reversals.

PACS. 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 02.50.Ey Stochastic processes – 87.10.+e General theory and mathematical aspects

1 Introduction

Recently, there has been increasing interest in studying the noise-induced transport of Brownian particles for systems with a spatially periodic potential field. It has been shown that the asymmetry of the potential [1,2], the asymmetry of the driving noise [3], and the input signal [4] are ingredients for the transport. These subjects were motivated by the challenge to explain the lack of direction of transport in biological systems, and several models have been proposed to describe a muscle's contraction [5–7], or the asymmetric polymerization of actin filaments responsible for cell mobility [1].

The rectification of noise leading to unidirectional motion in ratchet systems has been an active field of research over the last decade. In these systems directed Brownian motion of particles is induced by nonequilibrium noise in the absence of any net macroscopic forces and potential gradients. Several physical models have been proposed: rocking ratchets [8], flashing ratchets [9], diffusion ratchets [10], correlation ratchets [11], etc. In all these studies the potential is taken to be asymmetric in space. It has also been shown that one can obtain a unidirectional current in the presence of spatially asymmetric potentials. For these nonequilibrium systems an external random force should be asymmetric in time or the presence of space dependent mobility is required.

The study of current reversal phenomena has given rise to much research activity on its own. The motivation being the possibility of new particle separation devices superior to existing methods such as the electrophoretic method for particles of micrometer scale [12]. It is known that current reversals in ratchet systems can be engendered by changing various system parameters [13,14], including the flatness parameter of the noise [15], the correlation time of nonequilibrium fluctuations [16], the temperature in multinoise cases [17], the power spectrum of the noise source [18], the shape of the potential [19], the number of interacting particles per unit cell [20] and the mass of the particles [21]. In this paper, we study the current of a Brownian particle in a periodic symmetric potential in the presence of an asymmetric unbiased fluctuation and the inhomogeneous friction and show when the current reversals occur.

2 The current in an inhomogeneous system

The Brownian dynamics of the overdamped particle moving under the influence of a symmetric potential $V_0(x)$ and subject to a space dependent friction coefficient $\gamma(x)$ and asymmetric unbiased fluctuations at temperature T , is described by the Langevin equation (11)

$$\frac{dx}{dt} = -\frac{V_0'(x) - F(t)}{\gamma(x)} - k_B T \frac{\gamma'(x)}{[\gamma(x)]^2} + \sqrt{\frac{k_B T}{\gamma(x)}} \xi(t), \quad (1)$$

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where $\xi(t)$ is randomly fluctuating Gaussian white noise with zero mean and correlation: $\langle \xi(t)\xi(t') \rangle = 2\delta(t-t')$. Here $\langle \dots \rangle$ denotes an ensemble average over the distribution of the fluctuating forces $\xi(t)$. The primes in equation (1) denote the derivative with respect to the space variable x . It should be noted that equation (1) involves a multiplicative noise with an additional temperature dependent drift term. This term turns out to be important in order for the system to approach the correct thermal equilibrium state. We take $V_0(x) = V_0(x + 2n\pi) = -\sin(x)$, n being any natural integer. Also, we take the friction coefficient $\gamma(x)$ to be periodic: $\gamma(x) = \gamma_0(1 - \lambda \sin(x + \phi))$, where ϕ is the phase difference with respect to $V_0(x)$. The evolution of the probability density for x is given by the Fokker-Planck equation (1)

$$\frac{\partial P(x, t)}{\partial t} = \frac{\partial}{\partial x} \frac{1}{\gamma(x)} \left[k_B T \frac{\partial P(x, t)}{\partial x} + (V_0'(x) - F(t))P(x, t) \right] = -\frac{\partial j}{\partial x} \quad (2)$$

where j is the probability current and it can be expressed as follows:

$$j(x, t) = -\frac{1}{\gamma(x)} \left[(V_0'(x) - F(t)) + k_B T \frac{\partial}{\partial x} \right] P(x, t). \quad (3)$$

If $F(t)$ changes very slowly, there exists a quasi-stationary state. In this case, the average current of the particle can be solved by evaluating the constants of integration under the normalization condition and the periodicity condition of $P(x)$, and the current can be obtained and expressed as

$$j(t) = \frac{k_B T (1 - e^{2\pi F(t)/k_B T})}{\int_0^{2\pi} dy \exp(-F(t)y/k_B T) C(y)}, \quad (4)$$

where the space correlation function is given by

$$C(y) = \frac{1}{2\pi} \int_0^{2\pi} dx \gamma(x+y) \exp\left(-\frac{V_0(x) - V_0(x+y)}{k_B T}\right). \quad (5)$$

Considering that the external force $F(t)$ is slowly changing with the time, the average probability current J over the time interval of a period can be expressed by

$$J = \frac{1}{\tau} \int_0^\tau j(F(t)) dt. \quad (6)$$

where τ is the period of the driving force $F(t)$, which is assumed to be longer than any other time scale of the system in this adiabatic limit. Here we will consider a driving force with a zero mean $\langle F(t) \rangle = 0$, but which is asymmetric in time [22], as shown in Figure 1

$$F(t) = \frac{1+\varepsilon}{1-\varepsilon} F, \quad \left(n\tau \leq t < n\tau + \frac{1}{2}\tau(1-\varepsilon) \right), \quad (7)$$

$$= -F, \quad \left(n\tau + \frac{1}{2}\tau(1-\varepsilon) < t \leq (n+1)\tau \right). \quad (8)$$

In this case the time average current is easily calculated,

$$J = \frac{1}{2} \left[(1-\varepsilon)j\left(\frac{(1+\varepsilon)F}{1-\varepsilon}\right) + (1+\varepsilon)j(-F) \right]. \quad (9)$$

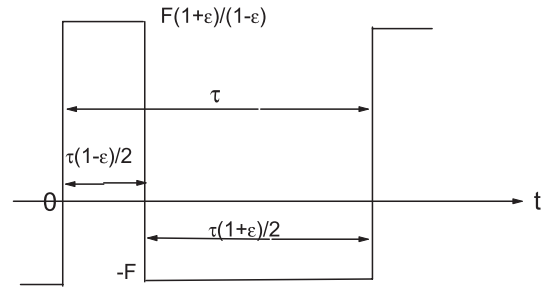


Fig. 1. The driving force $F(t)$ which preserves the zero mean $\langle F(t) \rangle = 0$, where the temporal asymmetry is given by the parameter ε .

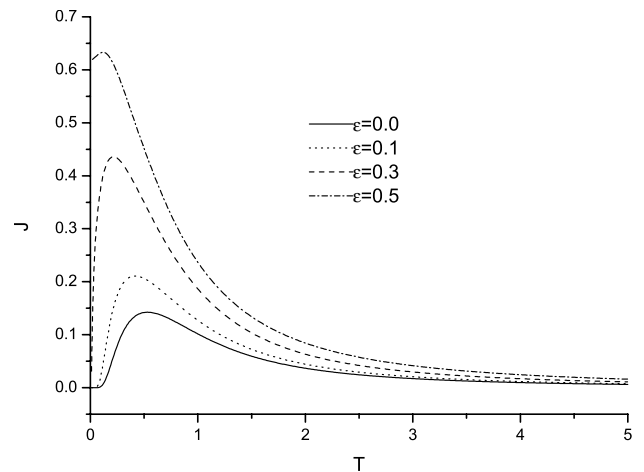


Fig. 2. The net current J versus temperature T for different values of ε ($\varepsilon > 0$). $\lambda = 0.9$, $\gamma_0 = 0.1$, $F = 0.5$ and $\phi = 1.3\pi$.

3 Results and discussion

We have calculated the average current about the motion of the Brownian particle in a periodic symmetric potential with asymmetric unbiased fluctuations.

The average current J is plotted in Figure 2 as a function of the temperature T for different asymmetry parameters (ε is positive). Here $\lambda = 0.9$, $\gamma_0 = 0.1$, $F = 0.5$ and $\phi = 1.3\pi$. The figure shows that the average current is a peaked function of the temperature. With an increasing intensity of the asymmetry parameter the maximal current increases, but the corresponding temperature at which the current takes the maximum is shifted to a lower temperature. For very high temperatures the current vanishes just like that of most of the thermal ratchet models [1-3]. It is obvious that no current reversals occur at the case $\phi > \pi$ and $\varepsilon > 0$. In Figure 3 we plotted the average current vs. temperature for different asymmetry parameters ($\varepsilon < 0$). The other parameters are the same as Figure 2. From Figure 3, we can see that when the asymmetry parameter is negative current reversal may occur ($\varepsilon = -0.3, -0.5, -0.7$) and the current vanishes for lower temperatures as well as higher temperatures.

In Figure 4, the plot of J versus F is shown for different asymmetry parameters (ε is positive), keeping λ , γ_0 , T and

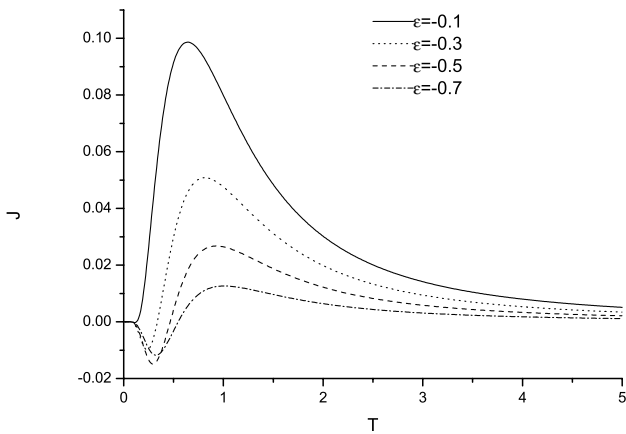


Fig. 3. The net current J versus temperature T for different values of ε ($\varepsilon < 0$) at The other parameters are the same as in Figure 2.

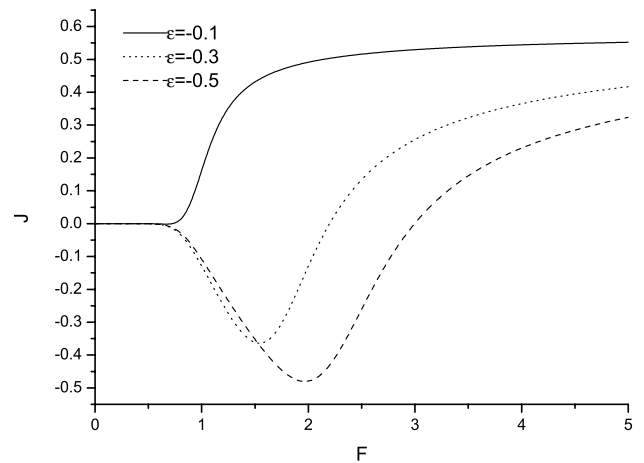


Fig. 5. The net current J versus rocking force F for different values of ε ($\varepsilon < 0$). The other parameters are the same as in Figure 4.

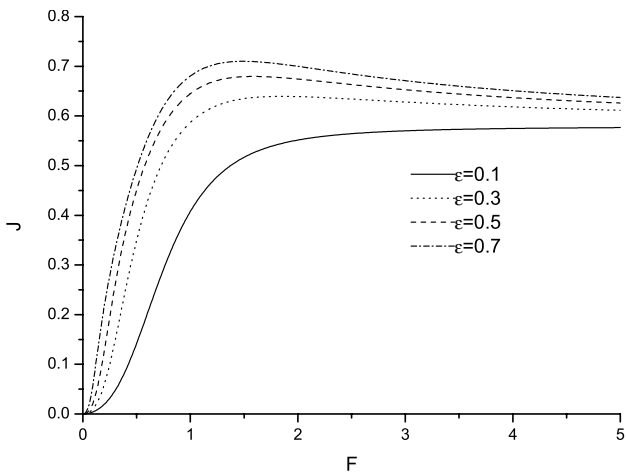


Fig. 4. The net current J versus rocking force F for different values of ε ($\varepsilon > 0$). $\lambda = 0.9$, $\gamma_0 = 0.1$, $T = 0.5$ and $\phi = 1.3\pi$.

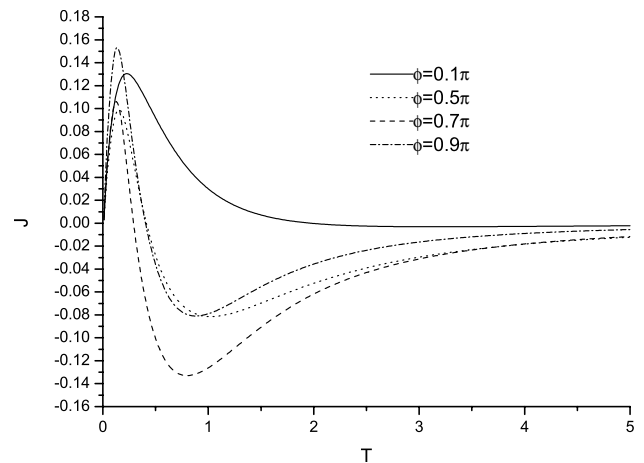


Fig. 6. The net current J versus temperature T for different values of ϕ . $\lambda = 0.9$, $\gamma_0 = 0.1$, $F = 0.5$ and $\varepsilon = 0.3$.

ϕ fixed at 0.9 , 0.1 , 0.5 and 1.3π respectively. With an increase of the asymmetry parameters, the current increases. For very large values of F , the current asymptotically goes to a positive constant value depending on the value ϕ , as was previously shown for the adiabatic case [14]. In the absence of space dependent friction ratchets where the currents saturate to zero in the same asymptotic regime. It is obvious that there are no current reversals in the case $\phi > \pi$, $\varepsilon > 0$. But the current reversals vs. F may occur (see Fig. 5) when the asymmetry parameters are negative ($\varepsilon = -0.3$, $\varepsilon = -0.5$). In the present case (Fig. 5) ε is chosen so that the current goes to negative at a lower temperature and positive at a higher temperature which guarantees that the current reversal occurs.

In Figure 6 we plotted J for various values of ϕ ($\phi < \pi$) at $\varepsilon = 0.3$ as a function of temperature T . Here $\lambda = 0.9$, $\gamma_0 = 0.1$ and $F = 0.5$. When $0 < \phi < \pi$ and $\varepsilon = 0.3$ ($\varepsilon > 0$), current reversal may occur and the current saturates to zero for high temperatures, namely at high temperatures the ratchet effect disappears.

In Figure 7 the average current J is plotted vs. F for $\phi = 0.5\pi$ and $\varepsilon = 0.3$. The other parameters are the same as in Figure 6. From Figure 7 we can see that the current reversal vs. F occurs for the case $\phi < \pi$, $\varepsilon > 0$. The current saturates to a negative constant for large values of F which is similar to Figure 4.

When the fluctuation is temporally asymmetric, its correlation properties in either direction are different, and a net current can arise even in the absence of a spatial asymmetry and a space dependent friction [22]. The positive asymmetric parameters induce a positive directional current while the negative parameters give a negative directional current. On the other hand, the phase difference between the friction coefficient and the symmetric potential is sensitive to the direction of the net current. Even in a symmetric potential and symmetric fluctuations [14] a net current can arise. The current tends to be positive in direction for $\phi > \pi$ and negative in direction for

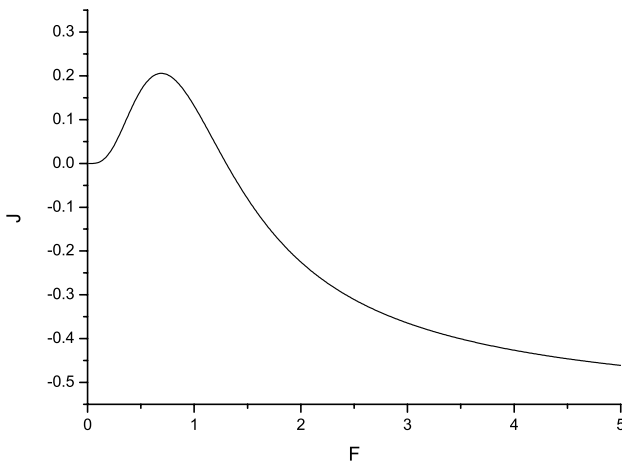


Fig. 7. The net current J versus rocking force F for $\phi = 0.5\pi$, $\lambda = 0.9$, $\gamma_0 = 0.1$, $T = 0.5$ and $\varepsilon = 0.3$.

$0 < \phi < \pi$. In fact, our ratchet contains these two driving factors. It is found that the current reversals may occur when a negative driving factor meets a positive driving factor: Case A ($\varepsilon < 0$ and $\phi > \pi$ see Fig. 3, Fig. 5), Case B ($\varepsilon > 0$ and $0 < \phi < \pi$ see Fig. 6, Fig. 7), while no current reversals occur when two negative (positive) driving factors meet: Case C ($\varepsilon > 0$ and $\phi > \pi$ see Fig. 2, Fig. 4), Case D ($\varepsilon < 0$ and $0 < \phi < \pi$).

In a word, in our symmetric potential ratchet, $\varepsilon\phi < 0$ is the necessary condition for current reversals, the particle never changes its moving direction under the condition of $\varepsilon\phi > 0$ and no current reversals occur under the condition of $\varepsilon\phi = 0$.

4 Summary and conclusion

In the present paper, the transport of a Brownian particle moving in a spatially symmetric potential in the presence of an asymmetric unbiased fluctuation is investigated. The current of the ratchet is discussed for different cases. We find that the mutual interplay between the opposite driving factors is the necessary term for current reversals. We find current reversal, both as a function of temperature as well as the amplitude of rocking force, when the force is adiabatic and the potential is symmetric.

The phenomena of current reversals may be of interest in biology, e.g., when considering the motion of macromolecules where the probable mechanism of vesicle transport inside eukaryotic cells was found to be the motion of proteins along a microtubule modeled as a ratchet. Myosin moves along actin filaments towards their plus extremity, and kinesins and dyneins move along tubulin filaments towards their plus and minus extremities respectively. It is well known that the two current reversals effect allows one pair of motor proteins to move simultaneously in opposite directions along the microtubule inside the eukaryotic cells. Several biological molecular motors, for instance kinesin

and non-claret disjunctional, belonging to the same superfamily of motor proteins move towards opposite ends of the microtubules. The ratchet mechanism was used for obtaining efficient separation methods of nanoscale objects, e.g., DNA molecules, proteins, viruses, cells, etc. To date, the feasibility of particle transport by man-made devices has been experimentally demonstrated for several ratchet types.

To summarize, it is remarkable that the interplay of asymmetric unbiased fluctuations, in homogeneous friction and thermal noise with spatially symmetric potential generates such a rich variety of cooperation effects such as current reversals with temperature as well as the rocking force.

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